INTRODUCTION TO MODELS OF ECONOMIC GROWTH:

The world today is a very different place than it was 200 years ago (see http://www.youtube.com/watch?v=jbkSRLYSooj). The total amount of economic activity has risen in part to just simply sustain – at some minimal level – a population more than seven times larger than it was in 1813. However, output and therefore real income have grown far more than this so that real per capita income is significantly greater worldwide. For example, here in the US we have the following:

![Graph showing RPCI US (2005 dollars) from 1750 to 2050.](image)

Real income per person is over 28 times greater now than it was at the beginning of the 19th century and in fact is almost three times higher than when I was born and 33% higher than it was in the early 1990s.

The questions of what forces promote growth, what determines whether or not the benefits of this growth are widely distributed or confined to a relative few, and the costs of growth are all part of development theory. What follows is my attempt to give you an overview of economic growth theory (explanations that economists have advanced to explain growth). It is by no means exhaustive but I hope clarifies some of the questions you might have even after having read our text.

**The classical period (1776 to the mid 19th century)**: 1776 was the year the father of modern economics’, Adam Smith published his book, *The Wealth of Nations* (WON). During this period most economists focused a great deal of their attention on the question of economic growth. As a result, all classical economists were development economists! However, other than one part of Smith’s WON, the bulk of attention was directed toward the limits to growth that agriculture presented. This is why 19th century English historian Thomas Carlyle dubbed economics “the dismal science”. Outside the hallowed halls of academia the monumental changes to the British economy that were taking place largely unnoticed. These changes were to prove, at least for a substantial period of time (from the early 19th century to the
present), that economic growth was not going to be held back by a limited supply of natural resources such as land.

**Marxism:** Karl Marx, the father of socialism, was strongly influenced by the free market classical economist David Ricardo (Ricardo, among other things, gave us the principle of comparative advantage which the doctrine of unregulated international trade is based on). Marx’s focus, like the classical economists such as Ricardo whom he had read and admired, was first and foremost on growth. However, unlike his classical forbearers, Marx argued that capitalism was a passing phase. To make a long story short, this scared the hell out of other economists who, of course, were from wealthy successful backgrounds and the beneficiaries of capitalism. Their response was to basically change the subject and focus on a key weak spot in classical and Marxian economics, the question of how prices were determined. The neoclassical period (roughly 1870 to 1930) gave birth to modern microeconomics. Much of what we learn in courses like ECON 202 and ECON 302 is thanks to the work carried out by economists during this period. As the details of how markets function were developed the subject of economic growth and development and the question of where the world was heading was largely forgotten.

**20th century growth theory:**

Keynes’s *General Theory* was published in 1936 and marks the birth of macroeconomics. Virtually all economists before Keynes accepted the notion that market economies would tend to move toward equilibrium at full employment but Keynes’s “theory of output as a whole” drew attention to the important role actual (effective) demand played in guaranteeing this happens. The birth of modern growth theory (the rebirth of growth theory) grew directly out of Keynes’s work. The two most influential figures in this period were Keynes’s former student and then colleague at Cambridge, Roy Harrod, and Russian-born American economist Evsey Domar. I favor Domar’s presentation and base the following on Domar’s work.

**Domar growth model.**

Imagine an economy with no government spending or taxation and no foreign trade. Assume that there is a capital stock of $32,480 billion, that there is 5% depreciation, that autonomous consumer spending is $1,559 billion, and that consumers’ mpc is equal to 0.75. There is investment spending of .05*32480 billion or $1,624 billion to begin with. Each $10,000 worth of capital can produce $4002.46 worth of output. With a capital stock of $32,480 billion this means that this economy’s capacity output is $13,000 billion. However with autonomous consumer spending of $1,559 billion, an mpc of 0.75, equilibrium real income, and investment spending of $1,624 billion, equilibrium real GDP is “only” $12,732 billion. Let’s say that in order to bring the equilibrium level of real income into line with this economy’s capacity the central bank of this economy lowers interest rates enough that investment increases by $67 billion. This, together with the spending induced by an mpc of .75 and autonomous consumer spending of $1,559 billion, creates the $268 billion in added demand needed to make $13,000 billion of production, spending, and earnings not just a possibility but a reality. For Keynes, this was basically the end of the story.
Working independently, what both Harrod and Domar noted was that in such a case as outlined above net investment spending has increased by $67 billion. All else equal, net investment raises the capacity of any economy. As the capital stock grows, production possibilities will expand. For there not be a return to unemployment and unused capacity demand has to grow still more. This can be achieved by still more investment spending but as added investment raises demand it also means an even higher level of net investment and still more growth in capacity. Like the Red Queen in Alice in Wonderland it takes all the running you can do, to keep in the same place!

Supply is increasing by \( \sigma I \) or \( \Delta Y = \sigma I_{\text{net}} \) (where \( \Delta Y \) = the increase in capacity, \( \sigma = \) the amount of output the economy can produce per unit of capital (remember net investment adds to capital stock and through \( \sigma \) adds to capacity) and \( I_{\text{net}} = I - \delta K \)) while demand is growing by \( \Delta I/(1 - mpc) \) or \( \Delta Y^D = \Delta I/(1 - mpc) \).

Once this process starts, to maintain balance over time \( \Delta Y^s \) has to equal \( \Delta Y^D \) or \( \sigma I \) has to equal \( \Delta I/(1 - mpc) \). This means that once you start the net investment ball rolling it has to keep rolling at a percentage rate equal to \( \sigma(1 - mpc) - \delta \).

With our numbers here (\( \sigma = .400246 \), mpc = .75 and \( \delta = .05 \) this means that net investment has to grow at a continuous rate of \( .400246*(1-.75)-.05 \) or about 5.01% a year. All else equal, to catch up to other countries, what a country would have to do was raise the rate of growth in its net investment spending. Once it has opened this Pandora’s box it would have to make sure that it maintained this growth in spending (no more nor any less) because otherwise it would face demand growing faster than supply or the opposite.

*A children’s book that was popular ages ago when children were literate!

**See algebra given below.
Graphically:

Capacity.

AE, Y

12,732

3183

12732

13,000

Y
Capacity before net investment

Capacity AFTER net investment

AE

AE_Y

12,732

12,732

13,000

13,026.82

Y
Note that investment has to grow by about 6.7 billion over the original 67 billion increase (see A above in figure 3) if the added capacity created by the first round of net investment is to be used along with full use of the capacity already there before net investment took place. The growth in investment is over and above the additional $67 billion and so total investment rises to 1697.794 billion in the next period. Think what this means as far as capacity in the period that follows! It means that still more growth in investment spending has to take place for that added capacity is to be absorbed. Connect this argument and the graphs to the following numbers if you can:

As noted, the moral of this story is that for an economy to grow continuously at a rate up to its potential there has to be sufficient growth in demand. In a sense, the Chinese understand this better than anyone. The aggressive promotion of exports is in part a realization that they need to make sure that the growth in demand is enough to justify the growth in capacity. It is use of this added capacity that, of course, provides employment and higher real income. At any rate, with the Harrod-Domar (H-D) model growth theory was reborn. It should also be noted that the timing for a renewed focus on growth and development could not have been better as first India (August 15th 1947) and then one country after another gained its independence from countries such as Great Britain that had, at least, temporarily gained an upper hand economically. There was a tremendous desire to think of policies that would help now economically disadvantaged countries catch up. The Harrod-Domar model was there to help newly independent governments and their supporters plan.

The Solow model: Economists have many annoying habits and many failings. One of the most remarkable is a penchant for looking at someone else’s work and attacking one simplifying assumption (however admittedly unrealistic) and introducing another simplifying assumption in its place. The “neoclassical” (new classical) economists who in response to the failings of classical and Marxian price theory are an example. Marx and classical economists assumed that the prices of things could be traced back to how much labor was used in an item’s production. Neoclassicals, rightly, tore this to shreds but assumed away growth and development. Robert Solow was guilty of this with respect to the Harrod-Domar model. He quite rightly took issue with one of the simplifying assumption employed in the H-D model; the assumption that there are constant marginal returns to capital. This is what setting $\sigma$ equal to 4002.46 per $10,000 worth of capital and keeping it there means. With diminishing marginal returns, equal increases in capital yield less and less of an increase in output the more capital that had already accumulated. The 4002.46 per $10,000 should fall more and more as an economy accumulates more capital. Both Harrod and Domar recognized that the assumption of constant marginal returns to capital was unrealistic but wanted to explore what happened to the Keynesian model (where sufficient demand is not guaranteed) once net investment had time to increase capacity. Solow proceeded to explore what diminishing marginal returns to capital would do to the H-D model but fell back on the pre-Keynesian idea that actual (effective) demand was always enough to absorb capacity. This model is outlined in the text, will be the focus of class handouts, and will be dealt with in class in detail this week and beyond so I needn’t repeat myself here. In light of H-D all that we must note at present is that Solow assumes away the very real problem that actual (effective) demand may not grow at a rate fast enough to fully
use added capacity. Demand failing to grow sufficiently fast will obviously discourage continued expansion. Why build more factories and create more jobs if the factory we just built is only running half the time? This is a very important point to keep in mind as we discuss trade (think China above) and development of the rural sector in the weeks to come but now on to Solow.

$$\text{Algebra of the Harrod–Domar model:}$$

$$\text{Derivation of the growth equation.}$$

From EC201 Know

$$AE = \text{Aggregate Expenditure} = \text{Total Spending} = C + I$$

Let

$$C = a + mpc \cdot Y$$

$$I = \text{Gross Investment}$$

so

$$AE = a + mpc \cdot Y + I$$

In equilibrium: $$Y = AE$$ (Let $$a = 0$$ for simplicity!)

$$\text{So } Y = mpc \cdot Y + I$$

$$Y - mpc \cdot Y = I$$

$$(1 - mpc) \cdot Y = I$$

$$Y = \text{Equilibrium } Y = \frac{I}{1 - mpc}$$

$$(1 - mpc) = s = I/Y$$

Harrod’s point that

$$\Delta Y = \Delta I s$$

Measures change in Aggregate Demand:

$$\Delta Y^d = \Delta I s$$

$$\Delta Y^s = \Delta \text{ in Aggregate supply}$$

when

$$\text{Net} = I - \delta \cdot K$$

where $$\delta$$ = Depreciation Rate, $$K$$ = Capital Stock
Over time $\Delta Y^D = \Delta Y^S$ for equilibrium to be maintained.

So $\frac{\Delta I}{I} = \sigma (1 - \delta K)$

Divide both sides by $I$ and multiply by $s$

$\Rightarrow \frac{\Delta I}{I} = s\sigma (1 - \delta K)$

$\frac{\Delta I}{I} = s\sigma - s\sigma \frac{\delta K}{I}$

$I = sY$ (Investment equals Savings if economy in equilibrium)

So $\frac{\Delta I}{I} = s\sigma - s\sigma \frac{\delta K}{sY}$

$1 = \frac{dY}{Y}$ so $\sigma \frac{\delta K}{Y} = 1$

Above equation (labeled A) reduces to

$\frac{\Delta I}{I} = s\sigma - s$

$\sigma = 1$ so can rewrite as

$\frac{\Delta I}{I} = s - s$

Not from here but $g = \frac{dY}{Y} = \text{Rate of growth in real income}$

Gran alfa = $\frac{\Delta I}{I}$ so $\ldots$
(Here comes the punchline!)
\[ g = \Delta Y = \frac{Y(t+1) - Y(t)}{Y(t)} = \frac{s}{\sigma} - \delta \]

Growth in real per capita income equals
\[ g^* = \frac{s}{\sigma} - \delta - n = g - n \]

where \( n \) = Rate of population growth

Can rewrite a little more precisely as
\[ g^* = \frac{s}{\sigma} - \delta - n \]
\[ \frac{g^*}{1+n} \]  
(B)

Assuming \( \delta \), \( \sigma \) and \( n \) exogenous or constant.

This implies that if I have a target rate of growth in per capita income
(Say I want it to double in 20 years)

\[ \frac{72}{20} = 3.6 = g^* \]

Can find \( \frac{I}{s} \) that will generate that
result algebraically by solving (B) above for \( s \).